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# Application of the Pinch Technique to Neutral Current Amplitudes and the Concept of the $Z$ Mass

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## ABSTRACT

The pinch technique (PT) is applied to neutral current amplitudes, focusing on the mixing problem. Extending recent arguments due to Papavassiliou and Pilaftsis, it is shown that the use of the PT self-energies does not shift the complex-valued position of the pole through order  $O(g^4)$ . This leads (to the same accuracy) to a simple interpretation of  $M_Z$ , the mass measured at LEP, in terms of the PT self-energies. It is pointed out that the PT approach provides a convenient and rather elegant formalism to discuss important neutral current amplitudes, such as those relevant to four-fermion processes and LEP2.

# 1 Introduction

The Pinch Technique (PT) is an algorithm that automatically re-arranges S-matrix contributions into self-energies, vertex and box diagrams which are separately  $\xi$ -independent and satisfy very desirable theoretical properties [1,2]. We emphasize that this re-arrangement occurs naturally *before* any calculation is performed (one-loop integrations or Dirac algebra) by virtue of the naive Ward identities satisfied by the tree level vertices of the theory. This fact is particularly transparent in the formulation of the PT in terms equal time commutators of currents [2]. A temporary drawback has been that so far the PT has been fully developed only at the one-loop level. Very recently, however, Papavassiliou and Pilaftsis (P-P) have been able to extend to higher orders certain aspects of the construction of the PT self-energies in charged current processes [3]. In particular, they have obtained the important result that the use of the PT self-energies does not shift the complex-valued position of the pole, as determined from the conventional self-energies. This opens the door for a number of significant developments. For instance, P-P have proposed to use their PT formalism to discuss resonant amplitudes involving unstable particles.

In section 2 of this paper we apply the PT to the phenomenologically important neutral current amplitudes, focusing on the mixing problem. In section 3 we extend the P-P argument to the neutral current case and show that the position of the complex-valued pole is not shifted through  $O(g^4)$ . This permits us to obtain a very simple theoretical interpretation of the  $Z$  mass, measured at LEP. Specifically, through  $O(g^4)$ ,

$$M_Z^2 = M_0^2 + \Re \hat{A}(M_Z^2) , \quad (1.1)$$

where  $\hat{A}$  is the PT transverse self-energy of  $Z$ , including the contributions of tadpoles and  $\gamma Z$  mixing effects that start in  $O(g^4)$ , and  $M_0$  is the gauge invariant combination of the bare  $Z$  mass and the tadpole counterterms. In Section 3 we also remind the reader of the theoretical difficulties that arise if, as it was done for a long time, one attempts to define the renormalized mass in terms of Eq.(1.1) with  $\hat{A}$  replaced by the conventional self-energy  $A$ . Eq.(1.1) tells us that, through  $O(g^4)$ ,  $M_Z$  can be identified with the zero of the real part of the inverse transverse propagator, provided that this is constructed

with the PT self-energies. In Section 3 we also point out that the PT approach provides a convenient and rather elegant formalism to discuss neutral current amplitudes such as those relevant to four-fermion processes and LEP2. An important advantage of this formalism is that it treats bosonic and fermionic contributions on an equal footing.

## 2 Application of the PT to neutral current amplitudes

In this section we discuss the application of the PT to the phenomenologically important neutral-current amplitudes, focusing on the mixing problem and restricting the discussion to one-loop order.

Fig. 1 depicts the 16 one-loop self-energy graphs in the Standard Model (SM) contributing to neutral current processes such as  $e^+e^- \rightarrow f\bar{f}$ , where  $f$  is a generic fermion distinct from  $e$ , and  $e^+e^- \rightarrow W^+W^-$ , which will be important for LEP2 studies. In the figures  $\chi$  is the unphysical Goldstone boson associated with  $Z$  and it is understood that the blobs and lines stand for the conventional self-energies and tree level propagators in the  $R_\xi$  gauges. Thus, for example, the photon propagator,  $i\Delta_\gamma^{\mu\nu}$ , is given by

$$i\Delta_\gamma^{\mu\nu} = -i[t^{\mu\nu} + \xi_\gamma \ell^{\mu\nu}]/q^2, \quad (2.1)$$

where  $t^{\mu\nu} \equiv g^{\mu\nu} - q^\mu q^\nu/q^2$  and  $\ell^{\mu\nu} \equiv q^\mu q^\nu/q^2$ , and the  $Z$  propagator,  $i\Delta_Z^{\mu\nu}$ , by

$$i\Delta_Z^{\mu\nu} = -i[g^{\mu\nu} - (1 - \xi_Z)q^\mu q^\nu \Delta_\chi]/(q^2 - M_0^2), \quad (2.2)$$

where  $\Delta_\chi \equiv (q^2 - \xi_Z M_0^2)^{-1}$ . We also have the equivalent expression

$$\Delta_Z^{\mu\nu} = U_Z^{\mu\nu} - (q^\mu q^\nu/M_0^2)\Delta_\chi, \quad (2.3)$$

where

$$U_Z^{\mu\nu} = -t^{\mu\nu}/(q^2 - M_0^2) + \ell^{\mu\nu}/M_0^2 \quad (2.4)$$

corresponds to the propagator in the unitary gauge.

In the application of the PT the first step is to replace the conventional self-energies in Fig.1 by their PT counterparts. This is done by combining the conventional amplitudes with the one-loop pinch parts from vertex and box diagrams. The result are new,  $\xi$ -independent self-energies, that satisfy very desirable theoretical properties. As the answer

is  $\xi$ -independent, a convenient short-cut is to work from the outset in the  $\xi_i = 1$  gauge. In that case there are no pinch parts emerging from the propagators and, therefore, when one considers four-fermion processes, there are no pinch parts at one loop arising from box diagrams. Once the self-energies in Fig.1 have been replaced by their PT counterparts, there remains the  $\xi$  dependence of the tree level propagators in the  $R_\xi$ . This dependence, however, can be shown to cancel by employing the elementary Ward identities satisfied by the PT self-energies [4]. Defining the self-energies of vector bosons, scalar bosons, and mixed vector-scalar bosons, as the corresponding one-particle irreducible Feynman diagrams multiplied by  $-i$ ,  $i$ , and 1, respectively, and denoting the PT self-energies by caret amplitudes, we have [4]

$$q^\mu \hat{\Pi}_{\mu\nu}^{\gamma\gamma} = q^\mu \hat{\Pi}_{\mu\nu}^{\gamma Z} = q^\mu \hat{\Pi}_\mu^{\gamma\chi} = q^\mu \hat{\Pi}_\mu^{\gamma H} = 0, \quad (2.5)$$

$$q^\mu \hat{\Pi}_{\mu\nu}^{ZZ} + M_0 \hat{\Pi}_\nu^{\chi Z} = 0, \quad (2.6)$$

$$q^\mu \hat{\Pi}_\mu^{ZH} + M_0 \hat{\Pi}^{\chi H} = 0, \quad (2.7)$$

$$q^\mu q^\nu \hat{\Pi}_{\mu\nu}^{ZZ} + M_0^2 \hat{\Pi}^{\chi\chi} = 0. \quad (2.8)$$

Employing these Ward identities it is straightforward to show that the remaining  $\xi$ -dependence in the propagators cancels and the self-energy graphs can be split into two groups corresponding to transverse and longitudinal self-energies, each group containing four amplitudes. Decomposing, for instance,

$$\hat{\Pi}_{\mu\nu}^{ZZ} = t_{\mu\nu} \hat{\Pi}_T^{ZZ} + \ell_{\mu\nu} \hat{\Pi}_L^{ZZ}, \quad (2.9)$$

where the subindices  $T$  and  $L$  mean “transverse” and “longitudinal”, the transverse and longitudinal amplitudes are depicted in Fig.2(a) and Fig.2(b), respectively. The propagators in Fig.2(a) are  $-i/(q^2 - M_0^2)$  and  $-i/q^2$ ; in Fig.2(b) they are  $i/M_0^2$  and  $-i/(q^2 - m_0^2)$  where  $m_0$  is the bare Higgs mass. We note that the decomposition involves only the physical particles,  $\gamma$ ,  $Z$ ,  $H$ , with  $\xi$ -independent propagators and self-energies. It is worth noting that the only surviving  $q^2 = 0$  poles are associated with the photon exchange diagram of Fig.2(a). For example,  $\hat{\Pi}^{\gamma Z} \propto q^2$  and  $q_\nu J_\gamma^\nu = 0$  which cancel the  $1/q^2$  singularity in the  $\gamma Z$  mixing diagrams in Fig.2(a). As  $q^2 \rightarrow 0$ , the sum of the  $ZZ$  diagrams

in Fig.2(a) and Fig.2(b) includes  $(q_\mu q_\nu/q^2)[\hat{\Pi}_L^{ZZ}(q^2) - \hat{\Pi}_T^{ZZ}(q^2)]$ . Recalling that  $\hat{\Pi}_{\mu\nu}^{ZZ}$  is regular as  $q^2 \rightarrow 0$ , we see from Eq.(10) that in this limit  $\hat{\Pi}_L^{ZZ}(q^2) - \hat{\Pi}_T^{ZZ}(q^2) \propto q^2$ , which cancels the  $1/q^2$  singularity.

P-P have proposed a method to construct iterated chains of PT self-energies [3]. This will be illustrated in the  $Z$  case in Section 3. For the moment we point out that the iteration of the transverse PT self-energies can be summed up by the same procedure employed for ordinary self-energies. Specifically, one inverts the matrix [5]

$$M = \begin{pmatrix} q^2 - M_0^2 - \hat{\Pi}_T^{ZZ} & -\hat{\Pi}^{\gamma Z} \\ -\hat{\Pi}^{\gamma Z} & q^2 - \hat{\Pi}^{\gamma\gamma} \end{pmatrix}. \quad (2.10)$$

The inverse matrix is

$$M^{-1} = \frac{1}{D(q^2 - \hat{\Pi}^{\gamma\gamma})} \begin{pmatrix} q^2 - \hat{\Pi}^{\gamma\gamma} & \hat{\Pi}^{\gamma Z} \\ \hat{\Pi}^{\gamma Z} & q^2 - M_0^2 - \hat{\Pi}_T^{ZZ} \end{pmatrix}, \quad (2.11)$$

where  $D \equiv q^2 - M_0^2 - \hat{\Pi}_T^{ZZ} - (\hat{\Pi}^{\gamma Z})^2/(q^2 - \hat{\Pi}^{\gamma\gamma})$ . The physical meaning of Eq.(2.11) becomes more transparent if we consider the matrix element  $(\hat{\Gamma}_Z, \hat{\Gamma}_\gamma)_\mu t^{\mu\nu} M^{-1} (\hat{\Gamma}'_Z, \hat{\Gamma}'_\gamma)_\nu^T$ , where  $\hat{\Gamma}_Z^\mu$ ,  $\hat{\Gamma}'_Z^\mu$ ,  $\hat{\Gamma}_\gamma^\mu$ ,  $\hat{\Gamma}'_\gamma^\mu$  represent appropriate vertex functions of  $Z$  and  $\gamma$  with external particles. One readily finds [6]

$$(\hat{\Gamma}_Z, \hat{\Gamma}_\gamma)_\mu t_{\mu\nu} M^{-1} \begin{pmatrix} \hat{\Gamma}'_Z \\ \hat{\Gamma}'_\gamma \end{pmatrix}^\nu = [\hat{\Gamma}_Z + \hat{\Gamma}_\gamma \frac{\hat{\Pi}^{\gamma Z}}{q^2 - \hat{\Pi}^{\gamma\gamma}}]^\mu \frac{t_{\mu\nu}}{D} [\hat{\Gamma}'_Z + \hat{\Gamma}'_\gamma \frac{\hat{\Pi}^{\gamma Z}}{q^2 - \hat{\Pi}^{\gamma\gamma}}]^\nu + \hat{\Gamma}_\gamma^\mu \frac{t_{\mu\nu}}{q^2 - \hat{\Pi}^{\gamma\gamma}} \hat{\Gamma}'_\gamma{}^\nu. \quad (2.12)$$

Eq.(2.12) can be interpreted as describing the propagation of a  $Z$  with dressed propagator  $-it_{\mu\nu}/D$  between the sources  $\hat{\Gamma}_Z^\mu$  and  $\hat{\Gamma}'_Z{}^\nu$  and of a  $\gamma$  with dressed propagator  $-it_{\mu\nu}/(q^2 - \hat{\Pi}^{\gamma\gamma})$  between  $\hat{\Gamma}_\gamma^\mu$  and  $\hat{\Gamma}'_\gamma{}^\nu$ . It is interesting to note that this last term contains the effect of the running of the electromagnetic coupling constant  $\alpha$  involving both its fermionic and bosonic contributions [2]. The remaining terms in Eq.(2.12) represent  $\gamma - Z$  mixing effects.

Considering now the longitudinal part of a four-fermion amplitude, we use  $\hat{\Pi}_L^{ZZ} = -(M_0^2/q^2)\hat{\Pi}^{\chi\chi}$ ,  $\hat{\Pi}^{ZH} = -(M_0/q^2)\hat{\Pi}^{\chi H}$  where  $\hat{\Pi}^{ZH}$  is defined by  $\hat{\Pi}_\mu^{ZH} = q_\mu \hat{\Pi}^{ZH}$ , and we have employed Eq.(2.7) and Eq.(2.8). Using also  $\partial_\mu J_Z^\mu = -M_0 J_\chi$ , where  $J_Z^\mu$  and  $J_\chi$  are

the fermionic operators coupled to  $Z$  and  $\chi$ , the self-energy matrix to be inverted is given by

$$L = \begin{pmatrix} q^2 - \hat{\Pi}_{\chi\chi} & \hat{\Pi}_{\chi H} \\ \hat{\Pi}_{\chi H} & q^2 - m_0^2 - \hat{\Pi}_{HH} \end{pmatrix} \Rightarrow L^{-1} = \begin{pmatrix} \hat{D}_{\chi\chi} & \hat{D}_{\chi H} \\ \hat{D}_{\chi H} & \hat{D}_{HH} \end{pmatrix}, \quad (2.13)$$

where  $m_0$  is the Higgs bare mass and,  $\hat{D}_{\chi\chi} = [q^2 - \hat{\Pi}_{\chi\chi} - \frac{\hat{\Pi}_{\chi H}^2}{q^2 - m_0^2 - \hat{\Pi}_{HH}}]^{-1}$ ,  $\hat{D}_{HH} = [q^2 - m_0^2 - \hat{\Pi}_{HH} - \frac{\hat{\Pi}_{\chi H}^2}{q^2 - \hat{\Pi}_{\chi\chi}}]^{-1}$ ,  $\hat{D}_{\chi H} = -\hat{\Pi}_{\chi H}[(q^2 - \hat{\Pi}_{\chi\chi})(q^2 - m_0^2 - \hat{\Pi}_{HH}) - (\hat{\Pi}_{\chi H})^2]^{-1}$ . Thus, in this part of the amplitude all longitudinal  $Z$ 's can be replaced by  $\chi$ 's, their unphysical scalar counterparts, for arbitrary values of  $q^2$ .

We next consider the one-loop vertex diagrams which provide pinch parts to the self-energies and also receive vertex-like pinch parts from the boxes. They are transformed into new  $\xi$ -independent expressions which satisfy a number of desirable properties. i) they are UV finite, ii) the form factors extracted from them and associated with the various electromagnetic and weak moments of the external particles are IR finite and well behaved for  $q^2 \rightarrow \infty$  (they respect perturbative unitarity) [7] and iii) the new one loop vertices satisfy their tree level Ward identities. Because of the WI

$$q^\mu \hat{\Gamma}_\mu^{Z\bar{f}f} + iM_0 \hat{\Gamma}^{\chi\bar{f}f} = (g/2c_W) \left[ (g_V^f + g_A^f \gamma_5) \hat{\Sigma}_f(p) - \hat{\Sigma}_f(p+q)(g_V^f - g_A^f \gamma_5) \right], \quad (2.14)$$

the graphs with the  $\chi\bar{f}f$  vertices cancel against longitudinal  $\xi$ -dependent contributions from  $Z$  exchange (cf. last term in Eq.(2.3)). In this way one is left with graphs that contain only physical particles. Finally we turn our attention to the one-loop box graphs. In the case of four fermion processes, the remainder of the box diagrams after the PT subtraction is their expression in the t' Hooft-Feynman gauge ( $\xi_i = 1$ ), which is UV finite. For arbitrary  $\xi_i$ , when the external fermions are considered massless, the  $ZZ$ ,  $Z\gamma$  and  $\gamma\gamma$  boxes are gauge independent while the  $WW$  box gives pinch parts only to the self-energies. If the external fermions have a finite mass, then the box graphs also contribute pinch terms to the vertices. For  $e^+e^- \rightarrow W^+W^-$ , the PT expression for the boxes does not coincide with the one in the  $\xi_i = 1$  gauge, but it is still UV convergent.

### 3 Residual terms in $\widehat{\Pi}_T^{ZZ}$ and the definition of the $Z$ mass.

As mentioned in Section 2, P-P have proposed the construction of chains of PT self-energies. To two and higher loops, they have shown that this requires not only the pinch parts from the available vertex graphs at the ends of the chain, but also additional pinch contributions from the one-particle irreducible self-energies of higher order than the ones contained in the chain. An important result of their analysis is the demonstration that the use of the PT self-energies does not displace the complex-valued position of the pole. In this section we extend the P-P argument to neutral-current amplitudes taking into account the effect of mixing. We restrict ourselves to terms up to  $O(g^4)$ , as this is sufficient for our purposes. After proving that the position of the complex pole is not displaced through  $O(g^4)$ , we show how this leads to Eq.(1.1), which provides a simple field-theoretical definition of the  $Z$  mass measured at LEP, and permits to carry out the conventional mass renormalization.

We begin by considering the two-loop diagrams depicted in Fig.3. Recalling [2]

$$\widehat{\Pi}_T^{ZZ} = \Pi_T^{ZZ}|_{\xi_i=1} - (q^2 - M_0^2)4g^2c_W^2I_{WW}(q^2), \quad (3.1)$$

$$\widehat{\Pi}_T^{\gamma Z} = \Pi_T^{\gamma Z}|_{\xi_i=1} - (2q^2 - M_0^2)2g^2c_Ws_WI_{WW}(q^2), \quad (3.2)$$

$$\widehat{\Pi}_T^{\gamma\gamma} = \Pi_T^{\gamma\gamma}|_{\xi_i=1} - (q^2 - M_0^2)4g^2s_W^2I_{WW}(q^2), \quad (3.3)$$

where  $s_W^2 = 1 - c_W^2$  is an abbreviation for  $\sin^2\theta_W$ , and

$$I_{WW}(q^2) = i\mu^{4-n} \int \frac{d^n k}{(2\pi)^n} \frac{1}{(k^2 - M_W^2) [(k+q)^2 - M_W^2]}, \quad (3.4)$$

the diagrams in Fig.3(a) are equal to

$$i(g/c_W)^2 J_Z^\mu t_{\mu\nu} J_Z^\nu / (q^2 - M_0^2)^2 \left\{ \left[ \Pi_T^{ZZ}|_{\xi_i=1} - (q^2 - M_0^2)4g^2c_W^2I_{WW}(q^2) \right]^2 / (q^2 - M_0^2) \right. \\ \left. + \left[ \Pi_T^{\gamma Z}|_{\xi_i=1} - (2q^2 - M_0^2)2g^2s_Wc_WI_{WW}(q^2) \right]^2 / q^2 \right\}. \quad (3.5)$$

On the other hand, the ordinary self-energies plus the pinch parts of Fig.3(b,c,d) proportional to  $J_Z^\mu t_{\mu\nu} J_Z^\nu$ , both evaluated in the  $\xi_i = 1$  gauge, give only

$$i \left( \frac{g}{c_W} \right)^2 J_Z^\mu \frac{t_{\mu\nu}}{(q^2 - M_0^2)^2} J_Z^\nu \left\{ \frac{1}{q^2 - M_0^2} \left[ \Pi_T^{ZZ}|_{\xi_i=1} \right]^2 + \frac{1}{q^2} \left[ \Pi_T^{\gamma Z}|_{\xi_i=1} \right]^2 - 4g^2c_W^2\Pi_T^{ZZ}|_{\xi_i=1}I_{WW} \right. \\ \left. - \frac{q^2 - M_0^2}{q^2} 4g^2c_Ws_W\Pi_T^{\gamma Z}|_{\xi_i=1}I_{WW} + (q^2 - M_0^2)4g^4c_W^4I_{WW}^2 + \frac{(q^2 - M_0^2)^2}{q^2} 4g^4c_W^2s_W^2I_{WW}^2 \right\}. \quad (3.6)$$

The additional pinch terms that should be added to the chain of Eq.(3.6) in order to convert it into the PT chain of Eq.(3.5) are readily obtained from their difference and are given by

$$R_{ZZ}^{(2)} = -4g^2 \left( c_W^2 \Pi_T^{ZZ}|_{\xi_i=1} + c_W s_W \Pi_T^{\gamma Z}|_{\xi_i=1} \right) I_{WW} + (q^2 - M_0^2) 12g^4 c_W^4 I_{WW}^2 + (3q^2 - 2M_0^2) 4g^4 s_W^2 c_W^2 I_{WW}^2, \quad (3.7)$$

where the common factor containing the currents and the two propagators has been omitted. We observe that  $R_{ZZ}^{(2)}$ , in contrast to the amplitudes between curly brackets in Eq.(3.5) and Eq.(3.6), contains no propagators and thus is of the same form as the two-loop one-particle irreducible graphs of the  $Z$  self-energy  $\hat{\Pi}_T^{(2)ZZ}$ . Following the P-P approach, one adds  $R_{ZZ}^{(2)}$  to Eq.(3.6) and subsequently subtracts the same amplitude from  $\hat{\Pi}_T^{(2)ZZ}$ .

For completeness, we also give the residual two-loop terms for the photon and the mixed self-energies :

$$R_{\gamma\gamma}^{(2)} = -4g^2 s_W^2 \Pi^{\gamma\gamma}|_{\xi_i=1} I_{WW} - 4g^2 c_W s_W \Pi_T^{\gamma Z}|_{\xi_i=1} I_{WW} + 12g^4 s_W^4 q^2 I_{WW} + 4g^4 c_W^2 s_W^2 (3q^2 - M_Z^2) I_{WW}^2, \quad (3.8)$$

$$R_{\gamma Z}^{(2)} = -2g^2 c_W s_W \left( \Pi^{\gamma\gamma}|_{\xi_i=1} + \Pi_T^{ZZ}|_{\xi_i=1} \right) I_{WW} - 2g^2 \Pi_T^{\gamma Z}|_{\xi_i=1} I_{WW} + 4g^4 c_W s_W [3q^2 - (1 + c_W^2) M_Z^2] I_{WW}^2. \quad (3.9)$$

Since the mid-eighties a number of authors have proposed the idea that the  $Z$  mass and width be defined in terms of  $\bar{s}$ , the complex-valued position of the  $Z$  pole [8,9,6]. Specifically,  $\bar{s}$  is the solution of

$$\bar{s} = M_0^2 + \Pi_T^{ZZ}(\bar{s}) + [\Pi_T^{\gamma Z}(\bar{s})]^2 / [\bar{s} - \Pi^{\gamma\gamma}(\bar{s})], \quad (3.10)$$

where the  $\Pi$ 's stand for the conventional self-energies. If one instead employs the PT self-energies, the corresponding pole position is given by

$$\hat{s} = M_0^2 + \hat{\Pi}_T^{ZZ}(\hat{s}) + [\hat{\Pi}_T^{\gamma Z}(\hat{s})]^2 / [\hat{s} - \hat{\Pi}^{\gamma\gamma}(\hat{s})]. \quad (3.11)$$



As Eq.(3.10) is gauge invariant, for simplicity we evaluate it in the  $\xi_i = 1$  gauge. Subtracting Eq.(3.10) from Eq.(3.11), we get

$$\hat{s} - \bar{s} = \hat{\Pi}_T^{ZZ}(\hat{s}) - \Pi_T^{ZZ}(\bar{s}) + \left( [\hat{\Pi}^{\gamma Z}(\bar{s})]^2 - [\Pi_T^{\gamma Z}(\bar{s})]^2 \right) / \bar{s} + \dots, \quad (3.12)$$

where henceforth it is understood that the conventional self-energies are evaluated in the  $\xi_i = 1$  gauge and the ellipsis represent terms of  $\mathcal{O}(g^6)$  and higher. From Eq.(3.1) it is easy to see that  $\hat{s} - \bar{s}$  is of order  $\mathcal{O}(g^4)$  or higher. Therefore, in Eq.(3.12) we can replace  $\hat{\Pi}_T^{ZZ}(\hat{s}) \rightarrow \hat{\Pi}_T^{ZZ}(\bar{s})$  with an error of  $\mathcal{O}(g^6)$ . Decomposing the difference  $\hat{\Pi}_T^{ZZ}(\bar{s}) - \Pi_T^{ZZ}(\bar{s})$  into one and two-loop parts, we have

$$\left[ \hat{\Pi}_T^{ZZ}(\bar{s}) - \Pi_T^{ZZ}(\bar{s}) \right]_{(1)} = -(\bar{s} - M_0^2)4g^2c_W^2I_{WW}(\bar{s}) = -4g^2c_W^2\Pi_T^{ZZ}(\bar{s})I_{WW}(\bar{s}), \quad (3.13)$$

where we have used Eq.(3.1) and Eq.(3.10). Furthermore

$$\left[ \hat{\Pi}_T^{ZZ}(\bar{s}) - \Pi_T^{ZZ}(\bar{s}) \right]_{(2)} = \left( \Pi_T^{ZZ}(\bar{s}) \right)_{(2)}^P, \quad (3.14)$$

where  $\left( \Pi_T^{ZZ}(\bar{s}) \right)_{(2)}^P$  is the two-loop pinch part to be added to the conventional self-energy in order to convert it into its PT counterpart. Extending the P-P prescription to neutral currents, this amplitude is of the form

$$\left( \Pi_T^{ZZ}(\bar{s}) \right)_{(2)}^P = C_1(q^2 - M_0^2)V_2^P + C_2(q^2 - M_0^2)^2B_2^P - R_{ZZ}^{(2)}, \quad (3.15)$$

where  $V_2^P$  and  $B_2^P$  are two-loop pinch parts from vertex and box diagrams ( $C_1$  and  $C_2$  are just constants) and  $R_{ZZ}^{(2)}$  is given in Eq.(3.7). Setting  $q^2 = \bar{s}$ , it is clear that the first two terms in Eq.(3.15) contribute to  $\mathcal{O}(g^6)$ . Thus we have

$$\begin{aligned} \hat{\Pi}_T^{ZZ}(\bar{s}) - \Pi_T^{ZZ}(\bar{s}) &= -4g^2c_W^2\Pi_T^{ZZ}(\bar{s})I_{WW} - R_{ZZ}^{(2)} \\ &= 4g^2c_Ws_W\Pi_T^{\gamma Z}(\bar{s})I_{WW} - 4g^4c_W^2s_W^2I_{WW}^2\bar{s} + \dots \end{aligned} \quad (3.16)$$

On the other hand,

$$\begin{aligned} [\hat{\Pi}^{\gamma Z}(\bar{s})]^2 - [\Pi_T^{\gamma Z}(\bar{s})]^2 &= [\Pi_T^{\gamma Z}(\bar{s}) - 2g^2c_Ws_W(2\bar{s} - M_0^2)I_{WW}]^2 - [\Pi_T^{\gamma Z}(\bar{s})]^2 \\ &= -4g^2c_Ws_W\Pi_T^{\gamma Z}(\bar{s})I_{WW}\bar{s} + 4g^4c_W^2s_W^2I_{WW}^2\bar{s}^2 + \dots \end{aligned} \quad (3.17)$$

Combining Eqs.(3.12), (3.16), and (3.17) we find that the contributions of Eqs.(3.16) and (3.17) cancel ! Therefore,

$$\hat{s} - \bar{s} = \mathcal{O}(g^6) \quad (3.18)$$

Thus, in analogy with the P-P results, we find that through  $\mathcal{O}(g^4)$  the use of the PT self-energies does not displace the pole position.

Defining

$$\hat{A}(s) = \hat{\Pi}_T^{ZZ}(s) + [\hat{\Pi}^{\gamma Z}(s)]^2/[s - \hat{\Pi}^{\gamma\gamma}(s)] , \quad (3.19)$$

writing  $\hat{s} = \hat{m}_2^2 - i\hat{m}_2\hat{\Gamma}_2$  and taking the difference between Eq.(1) and the real part of Eq.(3.11), we obtain

$$M_Z^2 = \hat{m}_2^2 - \Im m\hat{A}'(\hat{m}_2^2)\hat{m}_2\hat{\Gamma}_2 + \dots \quad (3.20)$$

where  $\hat{A}'(s) = d\hat{A}/ds$ . To one-loop order, only fermionic loops contribute to  $\Im m\hat{A}'(\hat{m}_2^2)$ . In the scaling approximation, in which very small terms of order  $m_b^2/M_Z^2$  are neglected,

$$\Im m\hat{A}'(\hat{m}_2^2) = \Im m\hat{A}(\hat{m}_2^2)/\hat{m}_2^2 = -\hat{\Gamma}_2/\hat{m}_2 . \quad (3.21)$$

Thus,

$$M_Z^2 = \hat{m}_2^2 + \hat{\Gamma}_2^2 + \dots \quad (3.22)$$

Recalling Eq.(3.18) and writing  $\bar{s} = m_2^2 - im_2\Gamma_2$ , we see that, through  $\mathcal{O}(g^4)$ ,  $M_Z^2 = m_2^2 + \Gamma_2^2 + \dots$ . It has been previously noted that, in terms of  $m_1^2 \equiv m_2^2 + \Gamma_2^2$  and  $\Gamma_1/m_1 \equiv \Gamma_2/m_2$ , the resonant  $Z$  amplitude exhibits the  $s$ -dependent Breit-Wigner resonance employed in the LEP analysis [9]. Thus the PT mass,  $M_Z$ , defined by Eq.(1), can be identified with  $m_1$ , and therefore with the  $Z$  mass measured at LEP. We recall that a similar identification is not consistent if, instead of  $\hat{A}$ , one inserts the conventional self-energy  $A(M_Z^2)$  in Eq.(1). In that case, for values of the gauge parameter  $\xi < 1/4c_W^2$ ,  $\Im mA'(m_2^2)$  and, therefore  $M_Z^2$ , becomes gauge dependent [9]. The origin of this problem can be traced to the facts that the conventional self-energy is  $\xi$ -dependent and that the instability of the  $Z$  forces a shift from  $M_Z^2$  to the complex valued pole position  $\bar{s}$ . In fact, it was suggested in [2] that these problems could in principle be circumvented if somehow the conventional self-energy in the mass renormalization condition is replaced by a  $\xi$ -independent amplitude, in such a manner that the pole position  $\bar{s}$  is not shifted. We have shown that this is precisely what the PT does.

Inserting  $M_0^2 = M_Z^2 - \Re\hat{A}(M_Z^2)$  in the transverse  $Z$  propagator  $-i/(s - M_0^2 - \hat{A}(s))$  leads to  $-i/(s - M_Z^2 - \hat{A}(s) + \Re\hat{A}(M_Z^2))$ , the conventional mass renormalization for the

PT self-energy of the  $Z$ . In the resonance region, where  $s - M_Z^2 = \mathcal{O}(g^2 M_Z^2)$ , using the scaling approximation for  $\Im m \hat{A}(s)$ , the propagator can be expressed as

$$-i/[1 - \Re e \hat{A}'(M_Z^2)][s - M_Z^2 + is\Gamma_Z/M_Z] ,$$

which exhibits the characteristic  $s$ -dependent Breit-Wigner resonance employed in the LEP analysis. Here  $\Gamma_Z$  is the  $Z$  width evaluated through  $\mathcal{O}(g^4)$ . This expression demonstrates once more that  $M_Z$ , defined via Eq.(1), can be identified with the mass measured at LEP. The factor  $[1 - \Re e \hat{A}'(M_Z^2)]^{-1}$  can be perturbatively expanded in the amplitude's numerator where it can be combined with other radiative corrections. Off the resonance region, where  $s - M_Z^2 = \mathcal{O}(M_Z^2)$ , one can expand the propagator in powers of  $[\hat{A}(s) - \Re e \hat{A}(M_Z^2)]/(s - M_Z^2)$ . Alternatively, one can retain  $\hat{A}(s) - \Re e \hat{A}(M_Z^2)$  in the propagator's denominator, in which case one needs a renormalization prescription to eliminate the remaining UV divergences in  $\hat{A}(s) - \Re e \hat{A}(M_Z^2)$ . In the  $\bar{M}S$ -scheme, for instance, it is natural to retain  $(\hat{A}(s) - \Re e \hat{A}(M_Z^2))_{\bar{M}S}$ , where the  $\bar{M}S$  subscript indicates that the  $\bar{M}S$  renormalization has been carried out.

We emphasize that the PT self-energies are  $\xi$ -independent, treat the fermionic and bosonic contributions on an equal footing and, as explained before, do not displace the complex-valued position of the pole, as we showed explicitly up to  $\mathcal{O}(g^4)$ . The ultraviolet divergences reside in the one-loop self-energies and can be absorbed in the renormalization of the bare couplings. For  $|q^2| \gg M_Z^2$  the PT self-energies satisfy the RGE. Thus for large  $|q^2|$ , they can be interpreted as factors that transform the bare into running couplings. In summary, the PT approach provides a convenient and rather elegant framework to discuss important neutral current amplitudes.

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## 5 Figure Captions

**Figure 1** : One-loop self-energy diagrams for neutral currents amplitudes in the SM ( $R_\xi$  gauges). The self-energies are the conventional  $R_\xi$  amplitudes and the unoriented solid lines stand for the  $R_\xi$  bosonic propagators.

**Figure 2** : One-loop self-energy diagrams in the PT approach. The four diagrams in (a) correspond to the transverse PT self-energies of the vector bosons (proportional to  $t_{\mu\nu}$ ). The four diagrams in (b) involve  $\ell_{\mu\nu}\hat{\Pi}_L^{ZZ}$ ,  $\hat{\Pi}_\mu^{ZH}$ , and  $\hat{\Pi}^{HH}$ .

**Figure 3** : Two-loop chains of transverse PT self-energies and a class of related pinch parts in the PT approach. Only contributions proportional to the external currents  $J_Z^\mu$  and  $J_Z^\nu$  are shown.

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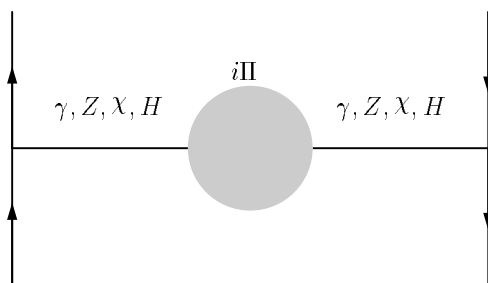


Figure 1

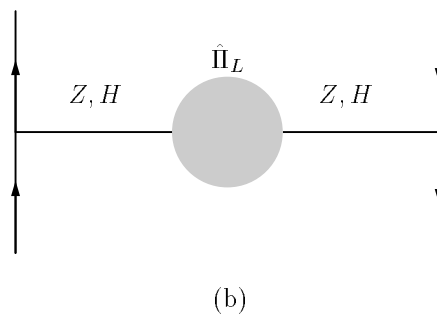
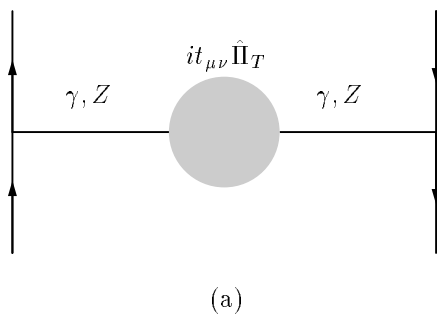


Figure 2

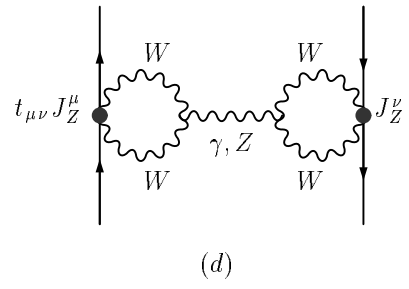
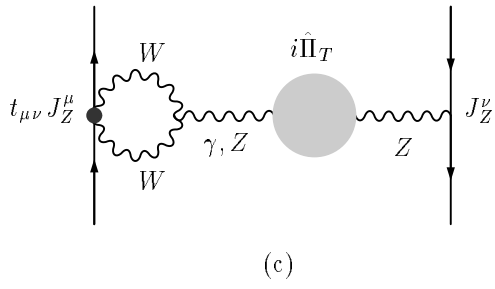
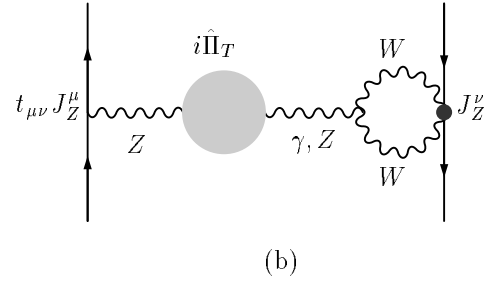
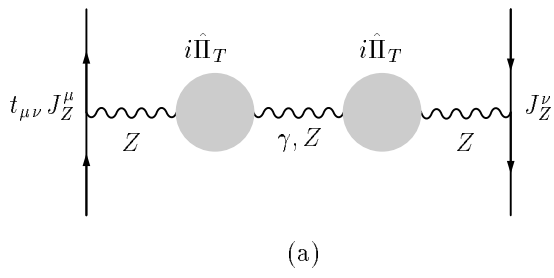


Figure 3